Cryptography

3 – Authentication and hash functions

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Today

User authentication

Hash function design

Message authentication

User authentication

Applications often need to ask users (or devices...) to identify themselves in order to know how to behave.

id : `Alice`

id : `Bob`

id : `Alice`

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Authentication factors

Obviously such an input needs to be **authenticated** (confirmed).

Authentication methods usually rely on **factors** such as:

- something the user *knows*,
- something the user *has*,
- something the user *is* (or a way he *behaves*).

Password authentication

Upon registration, every user provides (or is assigned) a password.

id : `Alice`

pw : `Ii(H48s`

id : `Bob`

pw : `secret`

. . .

Naive implementation

All valid pairs $(\mathrm{id}, \mathrm{pw})$ are stored by the service provider Sammy.

When a pair $(\mathrm{id}, \mathrm{pw}')$ is received, Sammy checks whether

$$\mathrm{pw}' = \mathrm{pw}.$$

Problem

An attacker with read access recovers all the passwords.

(Equivalently: need absolute trust in Sammy!)

Storing encrypted versions $E(k, \mathrm{pw})$ seems better…

…is it ? (hint: not really)

NB: *sending* encyrpted passwords on the communication channel is certainly a good idea, though

Solution

Use *one-way (lossy) encryption*

i.e. a **hash function**

$$H : \{0,1\}^* \longrightarrow \{0,1\}^n.$$

Examples:

MD5 (deprecated), SHA-1 (deprecated),

SHA-2, SHA-3, BLAKE2, Whirlpool, ...

Usage

A hash function turns everything into a fixed-length hex word.

```
from Crypto.Hash import MD5, SHA, SHA256

message = b"Hello"

print("init:", message)
print()
print("MD5 :", MD5.new(message).hexdigest())
print("SHA1:", SHA.new(message).hexdigest())
print("SHA2:", SHA256.new(message).hexdigest())
```

```
init: b'Hello'

MD5 : 8b1a9953c4611296a827abf8c47804d7
SHA1: f7ff9e8b7bb2e09b70935a5d785e0cc5d9d0abf0
SHA2: 185f8db32271fe25f561a6fc938b2e264306ec304eda518007d1764826381969
```

```
from Crypto.Hash import MD5, SHA, SHA256

message = b"hello"

print("init:", message)
print()
print("MD5 :", MD5.new(message).hexdigest())
print("SHA1:", SHA.new(message).hexdigest())
print("SHA2:", SHA256.new(message).hexdigest())
```

```
init: b'hello'

MD5 : 5d41402abc4b2a76b9719d911017c592
SHA1: aaf4c61ddcc5e8a2dabede0f3b482cd9aea9434d
SHA2: 2cf24dba5fb0a30e26e83b2ac5b9e29e1b161e5c1fa7425e73043362938b9824
```

Better password management

Sammy stores, for every valid user, a hash of their password:

$$(\mathrm{id}, h) \quad \text{with} \quad h = H(\mathrm{pw}).$$

Authentication:

Upon reception of $(\mathrm{id}, \mathrm{pw}')$, Sammy checks if

$$H(\mathrm{pw}') = h.$$

Requirement

The hash function should be **preimage resistant**:

given h, it must be computationally hard to find m such that

$$H(m) = h.$$

Attacks:

- brute force
- dictionary (precomputed)
- rainbow tables (space-time tradeoff)

Improvements

- **Salting**: store $(\text{id}, s, H(s \,\|\, \text{pw}))$ where s is random *salt*
- **Key stretching**: more generally, use a *key derivation function* to generate

$$k = K(s, \text{pw}) \quad \text{and store} \quad (\text{id}, s, k)$$

  where K is made *deliberately slow*

Examples: PBKDF2, Bcrypt, scrypt

$\Longrightarrow$ this is what should **always** be used in practice

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Cryptographic hash functions

Hash functions are useful for many things:

- id generation
- hash tables
- pattern detection
- serialization
- ...

but certain specific properties are required for their use in cryptography.

Requirements

- **determinism**: $m = m' \implies H(m) = H(m')$
- **uniformity**: every hash occurs with probability $1/2^n$
- **avalanche**: $m \approx m', m \neq m' \implies H(m) \not\approx H(m')$

  (exactly the inverse of **continuity**)

Things that should be hard

- given h, find m such that $H(m) = h$

  (**preimage resistance**)

- given m, find $m' \neq m$ such that $H(m') = H(m)$

  (**second preimage resistance**)

- find $m \neq m'$ such that $H(m) = H(m')$

  (**collision resistance**)

A textbook case: the story of SHA-1

- 1995: **Secure Hash Algorithm 1** standardized by NIST
- 2005: first "theoretical" collision attacks published
- 2010: collision complexity brought down to roughly 2^{60}

  Estimated cost of attack: 3 M$
- 2015: "the SHAppening" first practical attack demonstration

  Estimated cost of attack: 100 k$
- 2017: "SHAttered" first public collision
- 2019: Improved chosen prefix attack

The birthday problem

- generating $N > 2^n$ hashes $\Longrightarrow$ *certain* collision
- if N values are generated uniformly at random, the *probability* of a collision is

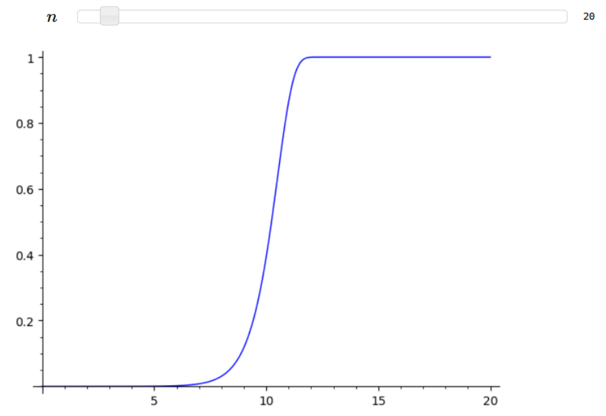
$$p = 1 - \prod_{k=0}^{N-1} \left(1 - \frac{k}{2^n}\right) \approx 1 - e^{-\frac{1}{2^n}\binom{N}{2}} \approx 1 - e^{-\frac{1}{2^{n+1}}N^2}$$

Example: The probability that 40 randomly chosen persons share a birthday is

$$\approx 1 - e^{-\frac{1}{365}\binom{40}{2}} \approx 88.2\%$$

NB: non-uniformity in the distibution of values only make collisions *more* probable

Collision probability as function of hash length



Birthday attack

One can show that the *average* number of values to be generated before a collision is found is approximately

$$\sqrt{\pi 2^{n-1}} \approx 1.25 \times 2^{\frac{n}{2}}.$$

Hence: a n-bit hash function provides $\leq \frac{n}{2}$ bits of security.

$\implies$ hashes need to be at least 256 bits long to provide 128 bits of security.

Pearson hash

An *insecure* construction

Divide the message m into k-bit blocks $(m_1, m_2, \dots)$

and choose a permutation σ of $\{0,1\}^k = [\![0, 2^k[\![$.

h = 0

for m_i in m:

$$h = \sigma(h \oplus m_i)$$

Nice, but specifying σ takes $k \cdot 2^k$ memory ...

Merkle-Damgård construction

Reuses the idea of Pearson hashing.

Pseudocode

$h = h_0$

for m_i in m:

$\quad h = F(h, m_i)$

where the *compression function* F is typically a simple operation iterated r times on the internal state (size s, divided into w-bit words)

Famous cryptographic hash functions

name	published	deprecated	n	k	s	w	r
MD5	1991	2000	128	512	512	32	64
SHA-1	1995	2005	160	512	160	32	64
SHA-2	2001	–	256 (224)	512	256	32	64
			512 (448)	1024	512	64	80
SHA-3	2012	–		...			

SHA-3 (Keccak)

Sponge construction

$(R, C) = (R_0, C_0)$

// absorption

for m_i in m:

$\quad (R, C) = F(R \oplus m_i, C)$

// then some more drying

eventually output R

Allows for certain freedom in choice of parameters

e.g. SHA3-224, SHA3-256, SHA3-384, SHA3-512, . . .

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Hash as checksum

Hash functions can be used to verity message integrity.

Alice: appends to a message m its hash $h = H(m)$.

Bob: verifies upon reception of (m, h) that $h = H(m)$.

(If not: transmission problem detected)

Example

$m = \text{You owe me 10 \$}$

$h = \texttt{c7b12b33fdd17399}$

$m_{\text{received}} = \text{You owe me 10 \$}$

$h_{\text{received}} = \texttt{c7b12b33fdd17399}$

$h_{\text{computed}} = \texttt{c7b12b33fdd17399}$

Ok !

Example (cont'd)

$m =$ You owe me 10 \$

$h =$ `c7b12b33fdd17399`

$m_{\text{received}} =$ You owe me 100 \$

$h_{\text{received}} =$ `c7b12b33fdd17399`

$h_{\text{computed}} =$ `08821af9be531f29`

Error !

But also...

$m =$ You owe me 100 \$

$h =$ `08821af9be531f29`

$m_{\text{received}} =$ You owe me 100 \$

$h_{\text{received}} =$ `08821af9be531f29`

$h_{\text{computed}} =$ `08821af9be531f29`

Ok ! ...

Problem

Even if H cannot be manipulated ...

anybody can compute a valid hash!

Double-edged sword:

- falsification
- repudiation

$\implies$ no authentication at all

Idea: encrypt the hash

Alice: appends to m its encrypted hash $h = E(k, H(m))$

Bob: upon reception of (m, h), checks whether $H(m) = D(k, h)$

Problem: since $H(m)$ and h are public, the secret key k is exposed...

Message authentication codes

Definition

A **MAC** consists of a *tag* function $\mathcal{K} \times \mathcal{M} \to \mathcal{T}$ as well as a *verification algorithm* that decides whether a particular MAC is valid for a given message.

- **Correctness**: every generated MAC should be valid
- **Forgery resistance** no one should be able to create a valid pair (m, t) without knowing the key.

From a hash function

Standard construction:

$$\mathrm{HMAC}(k, m) := H(\,(k \oplus \mathrm{opad}) \,\|\, H(\,(k \oplus \mathrm{ipad}) \,\|\, m)\,)$$

Alice: appends to m its tag $t = \mathrm{HMAC}(k, m)$

Bob: verifies upon reception of (m, t) whether $t = \mathrm{HMAC}(k, m)$

From a block cipher

Idea: Encrypt $m = m_1 \| \cdots \| m_\ell$ in CBC-mode with $\mathrm{IV} = 0$.

$$\text{CBC-MAC}(k, m) := c_\ell$$

$+$ additional precautions to prevent *extension attack*

Never reuse the same key for different purposes!

Authenticated encryption

Given a secure cipher + a secure MAC:

- **encrypt then MAC**: always ok
- encrypt and MAC: weakens encryption
- MAC then encrypt: ok in some cases

End remarks

- AE provides confidentiality, authentication, integrity, non-repudiation
- modern approach is to provide AE as a single primitive
- examples: OCB, EAX, EtM, GCM, CCM modes
- AE does not prevent *replay attacks* by itself

$\Longrightarrow$ Authenticated Encryption with Associated Data (AEAD) as IV should be used.